SOLUTION

CALCULUS II **QUIZ III** MATH-UA.0122-007

Write your solutions in steps.

1. (4 points) Approximate $\int_{-2}^{2} x^2\,dx$ by Simpson's Rule using $n = 8$
2. (3 points) Is $\int_{-2}^{-1} \frac{1}{x+1}\,dx$ convergent or divergent?
3. (3 points) Evaluate $\int_0^{\infty} \frac{1}{e^x}\,dx$

(1) $\Delta x = \frac{2-(-2)}{8} = \frac{1}{2} = 0.5$

$$\int_{-2}^{2} x^2\,dx \approx \frac{0.5}{3}\left((-2)^2 + 4\times(-1.5)^2 + 2\times(-1)^2 + 4\times(-0.5)^2 + 2\times 0^2 + 4\times 0.5^2 + 2\times 1^2 + 4\times 1.5^2 + 2^2\right)$$

$$= \frac{16}{3}$$

(2). Note $\frac{1}{x+1}$ has a vertical asymptote at $x = -1$.

$$\int_{-2}^{t} \frac{1}{x+1}\,dx = \int_{-2}^{t} \frac{1}{x+1}\,d(x+1) = \ln|x+1|\Big|_{-2}^{t} = \ln|t+1| - \ln 1 = \ln|t+1|$$

$$\int_{-2}^{-1} \frac{1}{x+1}\,dx = \lim_{t\to -1^-} \int_{-2}^{t} \frac{1}{x+1}\,dx = \lim_{t\to -1^-} \ln|t+1| = -\infty$$

So the improper integral diverges.

(3). $$\int_0^{\infty} \frac{1}{e^x}\,dx = \lim_{t\to+\infty} \int_0^t e^{-x}\,dx = \lim_{t\to+\infty} -e^{-x}\Big|_0^t = \lim_{t\to+\infty} -e^{-t} + e^0 = \lim_{t\to+\infty} -\frac{1}{e^t} + 1$$

$$= 1$$